Where Does Combustion-Formed Water Go? A Finite-Source, Climatological, Multi-Site Stochastic Framework for the Ground Return of Natural-Gas Combustion Water Vapor

Theoretical and computational framework with an illustrative application to a 360,000 Dth/day source near Delta, Utah and the Wasatch Front

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June 11, 2026

Abstract

Natural-gas combustion creates new water molecules by oxidizing fuel hydrogen and emits them as vapor in engine or turbine exhaust. The mass formed is fixed by stoichiometry; the open question is *where* and *how much* of that tagged combustion water first returns to the surface—through rain, snow, dew, frost, fog interception, engineered capture, or other condensation–deposition pathways—within any chosen accounting region, integrated over a realistic year of weather. We present a *theoretical and computational framework* for estimating this first-ground-return distribution. The framework is mass-conserving, phase-resolved, and concentration-aware: it couples a tagged two-phase water budget to a finite-source nonlinear saturation closure, stochastic transport, and competing removal hazards, and integrates deposition over a measured multi-year wind climatology at two contrasting Utah sites. For the representative western U.S. gas mixture, a 360,000 Dth/day source forms $\beta_{w,\mathrm{HHV}} \approx 41.8$ kg H_2O/Dth, i.e. $Q_w \approx 174$ kg/s, ≈ 12.2 acre-ft/day, $\approx 4{,}460$ acre-ft/yr; at this magnitude the tagged-vapor perturbation can rival local saturation deficits under cold, humid, stable, or shallow boundary layers, so transport is coupled to nonlinear saturation adjustment and concentration-dependent removal. We give both the rigorous pathwise hazard/survival formulation (the primary model) and a closed-form analytic limit used for interactive exploration, and demonstrate the framework's qualitative behavior for the dry, low-aerosol Delta high desert and the humid, aerosol-laden Salt Lake City / Wasatch Front. **The numerical deposition fractions reported here are illustrative scenario outputs of the analytic limit, not validated estimates**; site-specific stack characterization, a meteorological-ensemble simulation, and external trajectory benchmarks are required before they are used for water accounting, permitting, or policy. The robust qualitative message—that the large majority of combustion water is transported beyond the regional domain, with a strongly seasonal, precipitation-modulated minority depositing locally—does not depend on those calibrations.

Keywords: combustion water, natural gas, stoichiometry, tagged water vapor, two-phase tracer, Lagrangian particles, nonlinear condensation, saturation deficit, pathwise survival, wet deposition, dew, frost, aerosol–cloud interaction, wind rose, annual deposition footprint, mass conservation, acre-feet, Delta Utah, Wasatch Front.

Contents

Scope and status. This is a model-proposal paper. The source-term chemistry and the moist-air thermodynamics are standard and quantitative; the transport architecture is established stochastic-Lagrangian theory. Several deposition closures (condensate settling, dew/frost uptake, aerosol modulation) and the analytic combination of mechanisms are *order-of-magnitude parameterizations*, not calibrated schemes, and are flagged as such throughout. Numerical site results (§15) are illustrative. A companion numerical study would instantiate the framework with gridded meteorology (HRRR/ERA5), characterized stack geometry, and an ensemble, and would report calibrated fractions with uncertainty intervals.

1 Notation

Table 1 collects the principal symbols. The paper distinguishes *vapor* from *condensate* and *tagged* (combustion-formed) from *total* (combustion plus background) water; deposition is always written in tagged quantities.

Table 1: Principal notation. "Tagged" denotes combustion-formed water.

Symbol	Meaning
Q_w	combustion-formed water source, $\mathrm{kg\,s^{-1}}$
q_b	background vapor specific humidity, $\mathrm{kg\,kg^{-1}}$
q_t	tagged vapor specific humidity (perturbation), $\mathrm{kg\,kg^{-1}}$
$q_{\ell,t}$	tagged condensate (liquid/ice) specific humidity
$q = q_b + q_t$	total vapor controlling saturation
$q_s(T, p)$	saturation specific humidity over liquid ($q_{s,\ell}$) or ice ($q_{s,i}$)
$\Delta q_s = q_s - q_b$	pre-source saturation deficit
$q_+ = \max(0, q_b + q_t - q_s)$	total supersaturation excess
$\eta_t = q_t/(q_b + q_t)$	tagged fraction of condensing vapor
$c_{v,t}, c_{\ell,t}$	tagged vapor / condensate mass concentration, $\mathrm{kg\,m^{-3}}$
$R_c,\ R_{c,t} = \eta_t R_c$	total / tagged condensation rate, $\mathrm{kg\,m^{-3}\,s^{-1}}$
τ_c, τ_e	condensation / re-evaporation timescales
λ_k	first-return hazard for mechanism $k \in \{\mathrm{wet}, \mathrm{cond}, \mathrm{dew}, \dots\}$
$\gamma_p,\ P,\ W$	scavenging efficiency, precipitation rate, precipitable water
f_{dep}	condensate settling fraction reaching ground
v_{dew}	surface (dew/frost) deposition velocity, $\mathrm{m\,s^{-1}}$
φ	dimensionless aerosol loading
H_e	effective release height *above local ground* (§7)
σ_y, σ_z	Pasquill–Gifford crosswind/vertical spreads
$S_i(t)$	survival probability of particle i to time t
$w_s,\ f_{s,\theta},\ U_{s,\theta}$	season weight; sector frequency; sector mean speed

2 Purpose and modeling question

A natural-gas engine or turbine creates new water molecules by oxidizing the hydrogen in the fuel. Once these molecules leave the stack as vapor they join the atmospheric water cycle: they may condense inside the exhaust train, form transient plume droplets that re-evaporate, become cloud water, fall as rain or snow, deposit as dew or frost, be intercepted by fog, or leave any chosen accounting domain before returning to the surface.

The quantity of interest is a *first-ground-return distribution.* Let $\Omega \subset \mathbb{R}^2$ be a horizontal accounting domain and $B \subset \Omega$ a basin, county, watershed, or radial annulus. We seek

$$f_B = \frac{\text{mass of combustion-formed water whose first ground return lies in } B}{\text{mass of combustion-formed water emitted}}, \tag{1}$$

and the spatial deposition density $D(x, y)$ with $\int_B D\,\mathrm{d}x\,\mathrm{d}y$ the expected returned mass in B over an operating period. Western U.S. water volumes are reported in acre-feet, so all integrated returns are also given in acre-ft/yr.

This paper documents the governing chemistry and physics, the finite-source nonlinear closure, the deposition hazards and their combination, an aerosol modulation, the integration of deposition over a measured annual wind climatology, the multi-site parameterization, the two numerical realizations, and the validation and uncertainty program. It is the technical companion to a reference software implementation: a hybrid Lagrangian–Eulerian solver and an interactive analytic explorer whose governing equations are reproduced here.

3 Site definition and reference state

The primary modeled source is near Delta, Utah: $\phi_0 = 39.28846°$ N, $\lambda_0 = -112.46366°$ W, at the Delta-area station reference elevation z_s = 4,620.1 ft = 1,408.2 m (station USW00023162). The exact ground elevation should be replaced in implementation by a digital elevation model query. The International Standard Atmosphere barometric relation

$$p_s = p_0 \left(1 - \frac{L z_s}{T_0}\right)^{gM/(RL)}, \tag{2}$$

with $p_0 = 101325$ Pa, $T_0 = 288.15$ K, $L = 0.0065\,\mathrm{K\,m^{-1}}$, $g = 9.80665\,\mathrm{m\,s^{-2}}$, $M = 0.0289644\,\mathrm{kg\,mol^{-1}}$, $R = 8.3144598\ \mathrm{J\,mol^{-1}\,K^{-1}}$, gives $p_s \approx 85.5$ kPa.

Projection. A local tangent-plane projection maps geographic coordinates to local Cartesian (x, y) relative to a site origin (ϕ_0, λ_0),

$$x = R_E \cos(\phi_0)(\lambda - \lambda_0), \qquad y = R_E(\phi - \phi_0), \tag{3}$$

with $R_E = 6.371 \times 10^6$ m and angles in radians; the origin is a *site parameter* (§14). The tangent plane is accurate to $\lesssim 1\%$ in distance for the near-field and regional plume domains ($\lesssim 100$ km) over which the deposition field is resolved and rendered. *Regional accounting integrals* over the larger 500 km domain (§15) should use a geodesic area measure or an equal-area projection (Albers or Lambert conformal conic); we treat the 500 km figure as a "effectively-complete" accounting radius rather than a precisely projected boundary, and note that terrain and directional precipitation make any large fixed radius an approximation (§13.2).

4 Combustion-water source term: the chemistry

4.1 Stoichiometric formation

For a fuel species C_aH_b, complete combustion forms $b/2$ moles of water per mole of fuel, $C_aH_b + (a + \frac{b}{4})O_2 \to a\,CO_2 + \frac{b}{2}H_2O$. For a gas mixture with dry mole fractions z_i, molar masses M_i, and water coefficients $\nu_{w,i} = b_i/2$,

$$Y_{w/f} = \frac{\sum_i z_i \nu_{w,i} M_w}{\sum_i z_i M_i} \quad \left[\frac{\text{kg H}_2\text{O}}{\text{kg fuel}}\right], \qquad M_w = 18.0153 \text{ g/mol}. \tag{4}$$

For the representative western U.S. mixture $(z_{CH_4}, z_{C_2H_6}, z_{C_3H_8}, z_{N_2}, z_{CO_2}) = (0.90, 0.05, 0.02, 0.02, 0.01)$, $\sum_i z_i \nu_{w,i} \approx 2.03$ mol H_2O/mol dry gas and $Y_{w/f} \approx 2.050$ kg H_2O/kg fuel. The final implementation should use the delivered gas chromatograph and the billing heat-content basis.

4.2 Yield per unit fuel energy and the 360,000 Dth/day scenario

One decatherm (Dth) is one MMBtu ≈ 1.0550559 GJ, interpreted on a higher-heating-value (HHV) basis unless a contract specifies LHV. With molar higher heating values HHV_i,

$$\beta_{w,\text{HHV}} = \frac{\sum_i z_i \nu_{w,i} M_w}{\sum_i z_i HHV_i} \approx 39.6 \ \frac{\text{kg}}{\text{GJ}_{\text{HHV}}} = 41.8 \ \frac{\text{kg}}{\text{Dth}_{\text{HHV}}}, \tag{5}$$

using $\sum_i z_i HHV_i \approx 923.7$ kJ/mol. At $H_g = 360{,}000$ Dth day^{-1},

$$M_w \approx 360{,}000 \times 41.8 = 1.50 \times 10^7 \text{ kg day}^{-1}, \qquad Q_w = \tfrac{M_w}{86{,}400} \approx 174 \text{ kg s}^{-1}, \tag{6}$$

$$V_w \approx 12.2 \text{ acre-ft day}^{-1}, \qquad V_{w,\text{ann}} \approx 4{,}460 \text{ acre-ft yr}^{-1}. \tag{7}$$

The gross thermal input is $P_{th,\text{HHV}} \approx 4.40$ GW_{th}; paired with the reciprocating-engine coefficient $\alpha_{RICE} = 328$ kg H_2O/MWh [1], this is equivalent to ≈ 1.91 GW_e at $\approx 43.5\%$ HHV efficiency. On an LHV basis the yield is ≈ 46.3 kg/Dth, $Q_w \approx 193$ kg/s.

4.3 Net new water and engineered capture

Exhaust water also contains ambient humidity drawn in with combustion air; for hydrologic crediting the tagged source is the *newly formed* water Q_w only. An in-system capture fraction $\xi_{\text{cap}}(t) \in [0, 1]$ diverts part before release, so the atmospheric tagged source is $Q_{w,\text{atm}}(t) = [1 - \xi_{\text{cap}}(t)]\, Q_w(t)$, with captured water assigned to the source location.

5 Moist-air thermodynamics

Saturation vapor pressure over liquid water uses the Magnus form [2]

$$e_{s,\ell}(T) = 610.94 \exp\left(\frac{17.625\, T_c}{T_c + 243.04}\right) \text{ Pa}, \quad T_c = T - 273.15 \text{ [°C]}, \tag{8}$$

and *over ice* below freezing the corresponding Magnus coefficients,

$$e_{s,i}(T) = 611.21 \exp\left(\frac{22.587\, T_c}{T_c + 273.86}\right). \tag{9}$$

The saturation specific humidity is $q_s = \varepsilon e_s/[p - (1 - \varepsilon) e_s]$ with $\varepsilon = R_d/R_v = 0.62198$. For frost, snow, and mixed-phase processes a phase weight $\alpha_{\text{ice}}(T)$ interpolates $q_s = \alpha_{\text{ice}} q_{s,i} + (1 - \alpha_{\text{ice}}) q_{s,\ell}$, with $\alpha_{\text{ice}} = 0$ above 0°C, 1 below −40°C, and a smooth ramp between; the liquid form alone is the screening default. Moist density uses the virtual temperature $\rho_{\text{air}} = p/[R_d T(1 + 0.608q)]$. At the Delta reference state ($T = 280$ K, $p = 85{,}500$ Pa, $q_b = 3$ g/kg): $e_{s,\ell} \approx 990.4$ Pa, $q_{s,\ell} \approx 7.24$ g/kg, $\rho_{\text{air}} \approx 1.062$ kg m^{-3}, verified to machine precision against the simulation code.

6 Finite-source concentration and nonlinear coupling

6.1 Saturation excess and the tagged share

With q_b the background vapor and q_t the tagged perturbation, saturation is controlled by $q = q_b + q_t$ and the pre-source deficit is $\Delta q_s = q_s - q_b$. When $q_t \ll \Delta q_s$ the tag is a passive tracer and deposition scales linearly with Q_w; when $q_t \gtrsim \Delta q_s$ the emitted water can push the parcel to saturation. The nonlinear module activates wherever $q_t/\Delta q_s$ exceeds a threshold (~ 0.1). The screening supersaturation excess is

$$q_+ = \max\{0,\ q_b + q_t - q_s(T,p)\}, \tag{10}$$

total condensation occurs at rate $R_c = \rho_{\text{air}} q_+/\tau_c$, and—because the condensing pool is the *total* excess but only the tagged part is creditable to combustion—the tagged condensation rate is

$$R_{c,t} = \eta_t R_c = \frac{q_t}{q_b + q_t} \frac{\rho_{\text{air}} q_+}{\tau_c}. \tag{11}$$

Equation (11) is the tagging principle that all subsequent condensate deposition must use (§8). Small τ_c (10–60 s) is an aggressive local-condensation upper bound; larger τ_c (5–30 min) is conservative.

6.2 Energy-conserving saturation adjustment

Equation (10) fixes T and p, which is acceptable only as a screening diagnostic: condensation releases latent heat that warms the parcel and raises q_s, suppressing further condensation. The energy scale is not negligible here—fully condensing 174 kg/s would release 174×2.5 MJ/kg $\approx$ 435 MW—so saturated near-field cases require a conserved-enthalpy adjustment. Holding moist static energy (equivalently, liquid-water potential temperature) fixed,

$$c_p T + L_v q_v = \text{const}, \qquad q_v \le q_s(T,p), \qquad q_\ell = q_{\text{tot}} - q_v, \tag{12}$$

is solved (one Newton iteration on T suffices) so that condensation warms the parcel until $q_v = q_s(T,p)$; the tagged share (11) then partitions the resulting condensate q_ℓ. The fixed-T form (10) is retained for fast screening and far-field cases where $q_+ \ll q_s$ and the latent feedback is negligible.

6.3 Order-of-magnitude perturbation

A Gaussian-plume estimate of the ground-level perturbation, with image reflection (§7), is

$$\Delta q_t(x,0,0) = \frac{Q_w}{2\pi U \sigma_y \sigma_z \rho_{\text{air}}}\, 2\exp\left(-\frac{H_e^2}{2\sigma_z^2}\right), \tag{13}$$

which for $Q_w = 174$ kg/s, $U = 5$ m/s, $H_e = 30$ m, neutral spread is of order 1 g/kg at kilometer scale—comparable to cold-season deficits—confirming the finite-source correction is material. The sensitivity of (13) to the single-stack idealization is treated in §7.3.

7 Tagged-water transport and source geometry

7.1 Vertical coordinate convention

The vertical coordinate z is height *above local ground* ($z = 0$ at the surface), so the reflecting boundary and the Gaussian image term in (13) are referenced to $z = 0$, and the effective release height is

$$H_e = h_{\text{stack}} + \Delta h(t), \tag{14}$$

the physical stack height plus buoyant/momentum plume rise—*not* the station elevation z_s, which enters only through p_s and the saturation thermodynamics.

7.2 Eulerian and Lagrangian forms

With $c_{v,t}(\boldsymbol{r}, z, t)$ the tagged-vapor mass concentration, resolved wind $\boldsymbol{u}$, and diffusivity tensor $\boldsymbol{K}$,

$$\frac{\partial c_{v,t}}{\partial t} + \nabla \cdot (\boldsymbol{u} c_{v,t}) = \nabla \cdot (\boldsymbol{K} \nabla c_{v,t}) + S - R_{c,t} + R_{e,t} - \lambda_v c_{v,t}, \tag{15}$$

coupled to a tagged-condensate equation

$$\frac{\partial c_{\ell,t}}{\partial t} + \nabla \cdot (\boldsymbol{u}_\ell c_{\ell,t}) = \nabla \cdot (\boldsymbol{K} \nabla c_{\ell,t}) + R_{c,t} - R_{e,t} - \lambda_\ell c_{\ell,t}, \tag{16}$$

where $R_{e,t}$ is tagged re-evaporation and λ_v, λ_ℓ are the vapor- and condensate-phase removal hazards. This two-phase split enforces the distinction *condensation* $\neq$ *ground deposition*: condensate forms, may re-evaporate, and only the surviving settled or scavenged fraction deposits. A weighted particle ensemble realizes (15)–(16): particle i carries tagged mass m_i (vapor or condensate phase flag) and follows

$$\mathrm{d}\boldsymbol{X}_i = [\boldsymbol{u}(\boldsymbol{X}_i, t) + \nabla \cdot \boldsymbol{K}]\, \mathrm{d}t + \boldsymbol{B}_K\, \mathrm{d}\boldsymbol{W}_i, \quad \boldsymbol{B}_K \boldsymbol{B}_K^\top = 2\boldsymbol{K}, \tag{17}$$

in three dimensions with a reflecting ground boundary; the $\nabla \cdot \boldsymbol{K}$ drift enforces the well-mixed condition under inhomogeneous diffusivity [5].

7.3 Source geometry: distributed stacks

At 360,000 Dth/day the physical source is not a single stack but many engines or turbines distributed across a plant footprint, with individual stack heights, exhaust temperatures and velocities, plume rise, and possible building downwash. The source term is therefore a superposition

$$S(\boldsymbol{r}, z, t) = \sum_j Q_{w,j}(t)\, \delta(\boldsymbol{r} - \boldsymbol{r}_j)\, \delta(z - H_{e,j}(t)). \tag{18}$$

Because the saturation closure (10)–(11) is *nonlinear* in q_t, the tagged perturbations must be *superposed before* the closure is applied: $q_t = \sum_j q_{t,j}$, then form q_+ and $R_{c,t}$. Computing condensation stack-by-stack and summing would misrepresent the threshold. A single 30 m effective source can overstate near-field concentration if emissions are spread among many stacks, or understate it under downwash; source distribution is accordingly a first-order sensitivity (§17).

8 Deposition mechanisms (closed forms)

Each mechanism is a tagged flux in $\mathrm{kg\,m^{-2}\,s^{-1}}$; the rigorous pathwise combination is §9 and the analytic limit §10.

8.1 Precipitation scavenging with vertical overlap

Assuming the tag is mixed through the precipitating column, the column hazard is $\lambda_p = \gamma_p P/W$, with P the liquid-water-equivalent precipitation rate ($\mathrm{kg\,m^{-2}\,s^{-1}}$), W the total precipitable water over the precipitating column, and γ_p a mixing efficiency. A near-surface plume in a dry, stratified boundary layer is not automatically available to precipitating cloud water aloft, so the local hazard carries a vertical-overlap factor

$$\lambda_p(x, y, z, t) = \gamma_p \frac{P(x, y, t)}{W(x, y, t)} \chi_{\text{cloud}}(z, t)\, \chi_{\text{mix}}(z, t), \tag{19}$$

where χ_{cloud} restricts removal to the cloud layer and χ_{mix} encodes boundary-layer-to-cloud entrainment; setting $\chi_{\text{cloud}}\chi_{\text{mix}} = 1$ recovers the well-mixed column limit used in the analytic explorer (an upper bound on wet removal in stable conditions). Vertically integrating the Gaussian plume gives the column tagged water $W_t(x, y) = Q_w/[\sqrt{2\pi}\, U\sigma_y(x)] \exp(-y^2/2\sigma_y^2)$, so scavenging with upwind depletion at the column-mean hazard $\bar{\lambda}_p$ yields

$$D_{\text{wet}}(x, y) = \bar{\lambda}_p\, W_t(x, y)\, \exp\left(-\frac{\bar{\lambda}_p x}{U}\right). \tag{20}$$

This is mass-consistent: $\iint D_{\text{wet}}\, \mathrm{d}A = Q_w$ over infinite fetch, and the fraction depositing within downwind distance L is $f_{\text{wet}}(L) = 1 - \exp(-\bar{\lambda}_p L/U)$. Snow uses a phase-specific $\lambda_s = \gamma_s P_s/W$ with $q_{s,i}$.

8.2 Local condensate settling (tagged)

Condensate forms where $q_+ > 0$; per (11) only the tagged share is creditable. Writing the tagged column condensate as $\rho_{\text{air}}\, \eta_t\, q_+\, L_z$ over an explicitly defined effective vertical thickness L_z (the saturated-plume depth, *not* automatically σ_z; we take $L_z \sim \min(\sigma_z, z_i)$ with z_i the mixing depth), and a settling fraction f_{dep} that survives re-evaporation to reach the ground,

$$D_{\text{cond},t}(x, y) = f_{\text{dep}} \frac{\rho_{\text{air}}\, \eta_t(x, y)\, q_+(x, y)\, L_z(x)}{\tau_c}. \tag{21}$$

This corrects the untagged form: deposition is proportional to $\eta_t q_+$, not q_+, so background atmospheric water is not credited to combustion. The factor f_{dep} is the single most uncertain parameter in the model: visible plume droplets are typically small, re-evaporate readily, and have low settling velocities unless they grow, freeze, impact surfaces, or are embedded in fog/cloud [6]. The Monte Carlo solver resolves f_{dep} dynamically through (16) (form condensate, re-evaporate a fraction over τ_e, deposit the survivors); (21) is its steady screening surrogate.

8.3 Dew and frost surface uptake

With surface latent flux E (positive upward), the downward condensation flux is $C_s = \max\{0, -E\}$ and the physically grounded tagged dew/frost deposition is $G_{\text{dew}} = C_s\, q_t/q$, i.e. the surface condensation flux times the tagged fraction of near-surface vapor. The analytic explorer uses a coarse, surface-energy-balance-free surrogate: a deposition velocity acting on the *tagged near-surface specific humidity* $q_t^{sfc} \equiv \Delta q_t(x, y, z_1)$ at reference height z_1, modulated by a favorability factor,

$$D_{\text{dew}}(x, y) = v_{\text{dew}}\, \rho_{\text{air}}\, q_t^{sfc}(x, y)\, \mathcal{A}(\text{RH}, \text{class}), \quad \mathcal{A} = \frac{\max(0, \text{RH} - \text{RH}_0)}{1 - \text{RH}_0}\, s(\text{class}), \tag{22}$$

with $\mathrm{RH}_0 = 0.6$ and stability weight $s \in [0,1]$. The nominal $v_{\mathrm{dew}} \approx 5\times10^{-3}$ m/s is at the upper end of measured nocturnal water-vapor deposition velocities [9, 10]; it should be carried as an uncertain parameter with a broad prior (decades wide), and frost is the same term below freezing using $q_{s,i}$. This closure is explicitly illustrative.

9 Pathwise hazard/survival formulation

The physically correct first-deposition object is *pathwise*: each tagged parcel accumulates a survival probability against the time- and space-varying hazards along its own trajectory, and deposits stochastically with the instantaneous mechanism-specific rate. For particle i with trajectory $\boldsymbol{X}_i(s)$,

$$S_i(t) = \exp\left[-\int_0^t \sum_k \lambda_k\big(\boldsymbol{X}_i(s), s\big)\,\mathrm{d}s\right], \qquad \mathrm{d}P_{i,k}(t) = S_i(t)\,\lambda_k\big(\boldsymbol{X}_i(t), t\big)\,\mathrm{d}t, \tag{23}$$

and the deposited density for mechanism k is the expectation

$$D_k(\boldsymbol{r}) = \mathbb{E}\left[\int_0^T S(t)\,\lambda_k(\boldsymbol{X}(t), t)\,m(t)\,\delta\big((x,y) - \boldsymbol{X}_{xy}(t)\big)\,\mathrm{d}t\right]. \tag{24}$$

This formulation—which the Monte Carlo solver implements directly via the per-step draw $p_i = 1-\exp[-\lambda\Delta t]$ with mechanism chosen by $\lambda_k/\sum_j \lambda_j$—retains the timing, location, vertical overlap, meteorological dependence, and survival history of the actual tagged water, and is the *primary* model. It conserves mass automatically because survival can only decrease: $\sum_k \int \mathrm{d}P_{i,k} + S_i(\infty) = 1$.

10 Analytic limit: competing-removal combination

For interactive evaluation we use a closed-form analytic limit of (23)–(24) under the restrictive assumptions of a homogeneous domain, independent hazards, no vertical cloud-overlap limitation ($\chi = 1$), and no finite-domain anisotropy. Under independence the combined deposited fraction within the accounting domain is

$$f_{\mathrm{dep}}^{\mathrm{tot}} = 1 - \prod_{k\in\{\mathrm{wet,cond,dew}\}} (1 - f_k), \qquad f_k = \min\Big(1-\epsilon,\ \tfrac{\int D_k\,\mathrm{d}A}{Q_w}\Big), \tag{25}$$

the product of per-mechanism survival probabilities, with a small cap ϵ preventing $\log(0)$. The total deposited volume is $f_{\mathrm{dep}}^{\mathrm{tot}} Q_w$, apportioned by optical-depth share $\tau_k = -\ln(1-f_k)$ as $D_k^{\mathrm{tot}} = f_{\mathrm{dep}}^{\mathrm{tot}} Q_w\,\tau_k/\sum_j \tau_j$. By construction $f_{\mathrm{dep}}^{\mathrm{tot}} \le 1$. **This product form is an analytic toy limit, not the model:** it combines domain-integrated fractions after the fact and therefore discards the pathwise timing, vertical overlap, and directional/terrain dependence that (23) retains. It is used only where sub-second interactivity is required; quantitative results should come from the pathwise solver.

The instantaneous, phase-resolved budget that the solver maintains is

$$M_{v,t} + M_{\ell,t} + M_{i,t} + M_{\mathrm{dep},t} + M_{\mathrm{exp},t} + M_{\mathrm{cap},t} = M_{\mathrm{emit},t}, \tag{26}$$

tracking tagged vapor, liquid, ice, deposited, exported, and captured mass; its time-integrated form reduces to $M_{\mathrm{emit}} = M_{\mathrm{dep}} + M_{\mathrm{exp}} + M_{\mathrm{residual}} + M_{\mathrm{cap}}$. A 2×10^4-case sweep of the analytic limit (both sites, stability A–F, $\varphi \in [0,1]$, extreme humidity/precipitation) confirms the deposited fraction never exceeds unity (maximum ratio 1.000000) and that the per-mechanism split sums exactly to the total. Spatial maps render the per-mechanism flux sum as a relative density and are labeled as non-conserving visualizations.

11 Dispersion

Crosswind and vertical spreads $\sigma_y(x), \sigma_z(x)$ follow Pasquill–Gifford stability classes A–F via Briggs open-country (rural) forms $\sigma_y = ax/\sqrt{1+bx}$, $\sigma_z = cx(1+dx)^e$ [3, 4], with class-dependent constants. The near-stack singularity ($\sigma \to 0$) is excluded by restricting evaluations to $x \gtrsim 100$ m and a realistic effective release height; plume rise Δh should be parameterized from exhaust buoyancy and momentum in a calibrated run.

12 Aerosol modulation (uncertain, regime-dependent)

Atmospheric aerosols—mineral dust and anthropogenic particulate matter—act as cloud condensation nuclei (CCN) and ice nuclei (IN). Their net effect on precipitation and deposition is *not* monotonic and its sign is regime-dependent: more CCN can produce many smaller droplets and *suppress* warm-rain collision–coalescence; IN can *enhance* cold-cloud snow via the Bergeron process in some temperature/updraft regimes; and the outcome depends on cloud phase, supersaturation, aerosol composition, and cloud lifetime [6, 7, 8]. We therefore treat aerosol loading as an *uncertain sensitivity parameter*, not a guaranteed enhancement. A dimensionless loading $\varphi \in [0,1]$ modulates separate pathways through dimensionless gains whose priors are broad and *include values below one*:

$$\gamma_p \to \gamma_p \, g_p(\varphi, T, \text{phase}), \qquad\qquad f_{\text{dep}} \to f_{\text{dep}} \, g_s(\varphi, \text{droplet size}, \text{RH}, z_i), \tag{27}$$

with g_p, g_s centered near unity and spanning, say, 0.5–2. We do *not* lower the effective saturation deficit as a function of φ: cloud-droplet activation is a supersaturation process, and hygroscopic subsaturated uptake does not convert bulk vapor to precipitating condensate; the earlier $\Delta q_s \to \Delta q_s(1-k_n\varphi)$ device is withdrawn. The interactive explorer exposes a single monotone φ slider as a coarse "more-active-microphysics" control with the gains fixed positive; this is a *screening* choice and is flagged as such. Claims that aerosol effects are negligible at Delta or large along the Wasatch Front are *hypotheses requiring observational support* (PM, dust, AOD, and cloud-phase data): Delta-area playa dust and agricultural emissions may matter, and Wasatch winter inversions are often shallow and stable rather than efficient precipitation processors.

13 Annual deposition over a measured wind climatology

A single-condition deposition density $D(\boldsymbol{r}; \Theta, \theta, U)$ depends on the meteorological state $\Theta = (T, \text{RH}, P, \varphi, \dots)$, the wind bearing θ, and speed U. The annual footprint integrates this over the joint climatological distribution of season, wind direction, and *wind speed*.

13.1 Climatological decomposition

Partition the year into seasons $s \in \{\text{DJF}, \text{MAM}, \text{JJA}, \text{SON}\}$ with day-fraction weights w_s ($\sum_s w_s = 1$) and, within each season, a wind rose of $N_\theta = 16$ direction sectors with frequencies $f_{s,\theta}$, a calm fraction ($\sum_\theta f_{s,\theta} + f_{s,\text{calm}} = 1$), and a per-sector speed distribution $p_s(U \mid \theta)$. The annual deposition density is

$$D_{\text{ann}}(\boldsymbol{r}) = \sum_s w_s \sum_\theta f_{s,\theta} \int D\big(\boldsymbol{r}; \Theta_s, \theta, U\big) \, p_s(U \mid \theta) \, \mathrm{d}U. \tag{28}$$

The spatial result is a multi-lobed *rosette* about the source, with lobes along the prevailing sectors. The present analytic explorer evaluates (28) with the per-sector *mean* speed $U_{s,\theta}$ in place of the full integral; §13.2 quantifies the bias this introduces.

13.2 Direction-independence: scope and limits

Under *idealized* conditions—spatially homogeneous meteorology, an infinite or symmetric domain, no terrain, and no directionally varying precipitation or cloud fields—rotating the wind moves *where* water lands, not *how much*, so the domain-integrated deposited mass of a single plume, $\iint_\Omega D(\boldsymbol{r};\Theta,\theta,U)\,\mathrm{d}\boldsymbol{r} = \mathcal{M}(\Theta,U)$, is independent of bearing θ, and the annual budget collapses to a seasonal sum. *This independence is an idealization.* The real application has complex terrain, finite accounting domains, valley channeling, mountain barriers, seasonal and directional precipitation asymmetry, and basin boundaries; in that setting direction *does* change the integrated mass deposited inside a chosen region, and the correct annual budget retains the directional integral,

$$M_{\mathrm{ann}} = \sum_s w_s \sum_\theta f_{s,\theta} \int \mathcal{M}\big(\Theta_s,\theta,U\big)\, p_s(U \mid \theta)\,\mathrm{d}U, \tag{29}$$

rather than $\sum_s w_s \mathcal{M}(\Theta_s,\bar{U}_s)$. Two further cautions apply. First, using a season-mean speed $\bar{U}_s$ biases the budget because deposition fractions are nonlinear in U (Jensen's inequality): $\mathbb{E}[1 - e^{-\lambda L/U}] \neq 1 - e^{-\lambda L/\mathbb{E}[U]}$. Second, the 500 km accounting radius is itself direction- and terrain-dependent for export. The analytic explorer adopts the homogeneous, mean-speed simplification for interactivity and labels its budget accordingly; a calibrated run must use (29) with the speed distribution and gridded, terrain-aware meteorology.

13.3 Wind data and processing

The wind roses are built from ≈ 5 years of *sub-hourly* surface observations (direction and speed) from the Iowa Environmental Mesonet ASOS archive [14], which stores routine and special METAR reports at a typical 5–20 minute cadence (and denser during significant weather). At that cadence five years yields $\approx 5 \times 10^5$ valid records per station—consistent with the ≈ 11 records/hour we obtain—*not* the $\approx 4.4 \times 10^4$ that strictly hourly sampling would give. For the Delta site we use Milford (`MLF`), ≈ 50 km southwest, whose multi-year record is complete (the Delta municipal station's archive is too short); for the Wasatch site we use Salt Lake City International (`SLC`). Records are binned into the 16×4 (sector $\times$ season) grid. Because Milford is a ~ 50 km proxy across possibly different terrain channeling, a calibrated study should compare at least three wind sources at the source coordinate—`MLF` ASOS, a local Delta station if/when its record lengthens, and HRRR or ERA5 winds [16, 17]—and report directional and speed differences rather than adopt a single proxy. Temperature, relative humidity, and precipitation are taken from NOAA 1991–2020 monthly normals aggregated to seasonal means (Delta from Table 2; SLC from the corresponding NCEI normals [15]); the high-desert RH cycle is illustrative and is listed as a scenario assumption (Table 3). The processed climatology satisfies $\sum_s w_s = 1$ and rose closure for every site and season.

The resulting prevailing winds are physically faithful to the local terrain: Delta (MLF) prevails from the SSW ($\approx 202.5°$) at a year-weighted ≈ 6.3 m/s, consistent with channeling along the north–south Sevier Valley; Wasatch (SLC) prevails from the SE ($\approx 135°$) at ≈ 3.4 m/s with a high winter calm fraction, consistent with Salt Lake Valley down-valley drainage and frequent stable inversions.

14 Multi-site parameterization

The model is parameterized by a *site*: origin (ϕ_0,λ_0), elevation z_s and pressure p_s, seasonal climatology $(T,\mathrm{RH},P)_s$, wind rose $\{f_{s,\theta},U_{s,\theta}\}$, and baseline aerosol loading φ_0. The tangent-plane projection (3) is anchored at the site origin, so the rosette is computed and rendered about whichever

Table 2: Illustrative monthly climate covariates for the Delta station USW00023162 (NOAA 1991–2020 normals); the seasonal deficit uses p_s = 85.5 kPa and the *illustrative* high-desert RH cycle (Table 3).

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
T_{max}, °F	38.5	46.9	58.6	66.1	76.1	89.0	97.0	94.1	83.6	68.2	53.7	38.9
T_{min}, °F	13.2	19.4	26.2	31.7	39.6	48.9	57.6	54.8	44.7	32.3	22.1	13.7
Precip, in	0.8	0.7	1.0	0.9	1.2	0.5	0.5	0.7	0.9	0.8	0.6	0.9
Δq_s, g/kg	1.1	1.8	3.2	5.0	7.4	12.4	16.0	13.7	8.7	4.3	2.1	1.1

site is selected. Two contrasting sites are populated: **Delta, Utah** (z_s = 1,408 m, p_s = 85.5 kPa; dry high desert, seasonal RH ≈ 0.69 winter to ≈ 0.31 summer; assumed low baseline $\varphi_0 \approx 0.15$) and the **Salt Lake City / Wasatch Front** ($z_s \approx$ 1,288 m, $p_s \approx$ 86.3 kPa; more humid, wintertime particulate inversions; assumed higher baseline $\varphi_0 \approx 0.55$). The aerosol baselines are assumptions (Table 3), not measurements.

15 Illustrative results (unvalidated scenario outputs)

Status. The figures below are *scenario outputs of the analytic limit* under the assumptions of Table 3, not validated deposition estimates. They omit calibrated stack geometry, plume rise, gridded terrain-aware meteorology, the speed distribution (29), the vertical-overlap factor (19), and observation-constrained aerosol gains; they should be read as order-of-magnitude sensitivity illustrations with wide (currently unquantified) uncertainty.

For the 360,000 Dth/day source, neutral stability, H_e = 30 m, and baseline aerosol, the analytic annual limit integrated over each site's wind climatology gives, within an (approximate) 500 km accounting domain: **Delta** ≈ 15% deposited locally (≈ 650 acre-ft/yr), ≈ 85% exported, condensate settling and wet scavenging contributing comparably in the dry climate; **Wasatch** ≈ 36% deposited (≈ 1,600 acre-ft/yr), reflecting higher humidity, more precipitation, and a higher assumed aerosol baseline. These fractions credit only the tagged (combustion) share $\eta_t q_+$ of the condensate per (21); an earlier untagged screening over-credited background co-condensate and gave ≈ 20%/44%. Sweeping the (screening) aerosol loading at Delta from $\varphi = 0$ to $\varphi = 1$ raises the deposited fraction from ≈ 13% to ≈ 23% within the analytic cap. These percentages are sensitive to f_{dep}, τ_c, v_{dew}, the aerosol gains, the RH cycle, and the single-stack idealization—all high-sensitivity, weakly-constrained inputs (Table 3). The robust, calibration-independent conclusion is qualitative: *most* combustion water is exported beyond the regional domain, and local return is a strongly seasonal, precipitation-modulated minority, plausibly larger at the wetter Wasatch site than at Delta.

16 Computational realization

16.1 Hybrid Lagrangian–Eulerian Monte Carlo (primary)

The reference solver carries Lagrangian superparticles (position, mass, phase flag, age, status, counter-based RNG key) and an Eulerian grid (tagged vapor and condensate; background q_b, T, p; per-mechanism ground accumulators). Each timestep: release particles with $m_i \propto Q_{w,\text{atm}}\Delta t$ at the superposed sources (18); interpolate meteorology; advect and diffuse via Euler–Maruyama with ground reflection; bin to the grid for q_t; apply the saturation adjustment (12) and tagged condensation (11) into the condensate phase; re-evaporate a fraction over τ_e and settle the survivors;

Table 3: Key assumptions, status, and sensitivity of the illustrative results.

Assumption	Value	Basis	Sensitivity
Q_w	174 kg/s	fuel stoichiometry	low
H_e	30 m (single stack)	assumed	high
source geometry	one point source	assumed	high
τ_c	10–1800 s	heuristic	high
f_{dep} (settling)	unspecified / dynamic	heuristic	very high
γ_p, χ	$\gamma_p = 1$, $\chi = 1$	upper-bound limit	high
v_{dew}	5×10^{-3} m/s	upper-end literature	high
aerosol gains g_p, g_s	monotone, ≥ 1 (screening)	heuristic	high
φ_0	0.15 Delta, 0.55 Wasatch	assumed	high
RH cycle	illustrative high-desert	assumed	moderate–high
wind proxy	MLF ($\sim$ 50 km)	proxy station	moderate–high
accounting radius	500 km, tangent plane	approximate	moderate

draw wet/dew/condensate deposition by the pathwise survival (23); export particles leaving the domain; and verify the phase-resolved budget (26) each step. Counter-based random numbers make particle fate reproducible independent of batch order.

16.2 Analytic steady-state explorer (screening)

The closed forms (20)–(22), combined by (25) and integrated by (28), constitute an interactive ($<$ 1 s) screening limit of the Monte Carlo solver across sites, aerosol loadings, and source parameters. It adopts documented simplifications relative to the primary model—fixed-T q_+ (10), the product-formula combination (25), mean-speed and direction-independent budget, $\chi = 1$, single stack—each flagged where it is used. Quantitative results are the province of the pathwise solver.

16.3 First-deposition estimator

Given deposited records $(m_i, x_i, y_i, t_i, k_i)$ the gridded density is $\widehat{D}_k(\boldsymbol{r}) = \sum_{i:k_i=k} m_i K_h(\boldsymbol{r} - \boldsymbol{r}_i)$ and basin fractions $\widehat{f}_B = \sum_i m_i \mathbf{1}_{\boldsymbol{r}_i \in B} / (\sum_i m_i + M_{\text{exp}} + M_{\text{res}})$, with Monte Carlo error $SE \approx \sqrt{\widehat{f}_B(1 - \widehat{f}_B)/N_{\text{eff}}}$, $N_{\text{eff}} = (\sum_i m_i)^2 / \sum_i m_i^2$.

17 Validation and benchmarks

The implementation is checked against: (1) the source term, 360,000 Dth/day$\to$ 174 kg/s$\to$ 4,460 acre-ft/yr; (2) per-step, phase-resolved mass conservation (26); (3) the 1-D exponential downwind removal density $f(x) = (U\tau)^{-1} e^{-x/U\tau}$; (4) the 2-D advection–diffusion–removal kernel $D(x, y) = \int_0^\infty Q_w \frac{\lambda}{4\pi K_h t} \exp[-\lambda t - ((x - Ut)^2 + y^2)/4K_h t]\, dt$, which integrates to Q_w; (5) saturation activation; (6) a null test ($P = 0$, subsaturation $\to$ near-zero deposition); (7) Monte Carlo convergence $SE \propto N_{\text{eff}}^{-1/2}$; (8) the analytic-limit constraint that combined deposition never exceeds the source; (9) climatology integrity ($\sum w_s = 1$, rose closure, idealized direction-independence under rose rotation); and (10) aerosol sensitivity within the conservation cap. Three external/structural benchmarks are required before quantitative use and are part of the companion numerical study:

1. **Transport-envelope benchmark.** For passive (pre-removal) behavior, compare trajectory and dispersion envelopes against HYSPLIT or FLEXPART for selected weather cases [12,

11]; the water-specific removal is custom, but the transport should match before removal is activated.

2. **Phase-exchange conservation.** Verify (26) with condensation and re-evaporation active, tracking vapor, liquid, and ice tags through the saturation adjustment (12).

3. **Source-distribution sensitivity.** Run one 30 m point source; 10 distributed stacks; 100 engine exhausts; and cases with building downwash and with parameterized plume rise, superposing q_t *before* the saturation closure (18). This bounds the robustness of the "concentration-matters" conclusion.

18 Uncertainty quantification

Uncertainty propagates from five classes: source (gas composition, efficiency, heat rate, load, capture); release (stack height and *distribution*, plume rise, exhaust state, droplet survival f_{dep}); meteorology (winds and their proxy/interannual variability, boundary-layer height, humidity, cloud, precipitation timing and phase); deposition physics (γ_p, χ, τ_c, v_{dew}, and the aerosol gains g_p, g_s with priors spanning below and above unity); and domain/accounting (finite-domain and directional export, watershed boundaries, season weighting, first-return versus recycling). A Bayesian formulation computes $p(f_B \mid \mathcal{D}) = \int p(f_B \mid \theta, \mathcal{M})p(\theta \mid \mathcal{D})\, \mathrm{d}\theta$ and reports medians and credible intervals per region and mechanism; the illustrative results of §15 are the analytic limit at the prior modes and should be superseded by these intervals.

19 Conclusion

The mass of combustion-formed water is fixed by stoichiometry; its first ground return is a stochastic, seasonal, transport-and-deposition problem. This paper assembled a technical framework—combustion chemistry, moist-air thermodynamics, a finite-source energy-conserving saturation closure, a two-phase tagged transport budget, a pathwise hazard/survival deposition formulation with a closed-form screening limit, an uncertain regime-dependent aerosol modulation, and an annual footprint integrated over a measured multi-year wind climatology—realized as a hybrid Lagrangian–Eulerian solver with an interactive analytic explorer, and applied illustratively to two contrasting Utah sites. The representative 360,000 Dth/day source forms ≈ 174 kg/s (≈ 4,460 acre-ft/yr). The framework's calibration-independent message is that the majority of the formed water is transported beyond the regional domain, with a strongly seasonal, precipitation-modulated minority depositing locally. The specific local-return fractions reported here are illustrative; converting them into decision-grade estimates requires characterized source geometry, gridded meteorology with the directional/speed integral (29), observation-constrained deposition and aerosol parameters, and the external benchmarks of §17.

A Units and conversions

$$\beta_{w,\text{HHV}} \approx 41.8 \text{ kg}\,\text{Dth}^{-1}, \qquad Q_w \approx 174 \text{ kg}\,\text{s}^{-1}, \tag{30}$$

$$M_w \approx 1.50 \times 10^7 \text{ kg}\,\text{day}^{-1}, \qquad V_w \approx 12.2 \text{ acre-ft}\,\text{day}^{-1}, \tag{31}$$

$$V_{w,\text{ann}} \approx 4{,}460 \text{ acre-ft}\,\text{yr}^{-1}, \qquad 1 \text{ acre-ft} = 1233.48 \text{ m}^3. \tag{32}$$

A deposition flux D in $\mathrm{kg\,m^{-2}\,s^{-1}}$ converts to acre-ft/yr per km^2 by $D \times (86{,}400 \times 365.25) \times 10^6/[\rho_w\, 1233.48]$, $\rho_w = 999\ \mathrm{kg\,m^{-3}}$, a factor $\approx 2.56 \times 10^7$. Spread uniformly over A km^2, the annual mass corresponds to an equivalent depth $h_A \approx 5489/A\ \mathrm{mm\,yr^{-1}}$.

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